

Solutions to Maths workbook - 1 | Quadratic Equations

Level – 3

Daily Tutorial Sheet – 14

161.(C) The given equation be $x^2 - p(x+1) - c = 0 \Rightarrow x^2 - px - c = 0 \Rightarrow \alpha + \beta = p, \alpha\beta = -p - c$

$$\text{Now } \frac{\alpha^2 + 2\alpha + 1}{\alpha^2 + 2\alpha + c} + \frac{\beta^2 + 2\beta + 1}{\beta^2 + 2\beta + c} = \frac{(\alpha + 1)^2}{(\alpha + 1)^2 - 1 + c} + \frac{(\beta + 1)^2}{(\beta + 1)^2 - 1 + c}$$

[Using : $(1 + \alpha)(1 + \beta) = 1 + (\alpha + \beta) + \alpha\beta = 1 + p - p - c = 1 - c$]

$$\begin{aligned} &= \frac{(\alpha + 1)^2}{(\alpha + 1)^2 - (1 + \alpha)(1 + \beta)} + \frac{(\beta + 1)^2}{(\beta + 1)^2 - (1 + \alpha)(1 + \beta)} \\ &= \frac{\alpha + 1}{\alpha + 1 - 1 - \beta} + \frac{\beta + 1}{\beta + 1 - 1 - \alpha} = \frac{\alpha + 1}{\alpha - \beta} + \frac{\beta + 1}{\beta - \alpha} = \frac{\alpha + 1 - \beta - 1}{\alpha - \beta} = 1 \end{aligned}$$

Paragraph for questions Nos. 162 to 164

$$f(x) - (6 - x) = 0 = (x - 1)(x - 2)(x - 3)(x - 4)(x - 5)$$

Since polynomial is of degree 5.

$$\text{So, } f(x) = (x - 1)(x - 2)(x - 3)(x - 4)(x - 5) + (6 - x) = 0 \dots\dots (a_1, a_2, a_3, a_4, a_5)$$

$$f(6) = 120,$$

$$a_1 + a_2 + a_3 + a_4 + a_5 = 15$$

$$a_1 \cdot a_2 \cdot a_3 \cdot a_4 \cdot a_5 = 114$$

162.(A), 163.(A), 164.(C)

165. Here the equation is $\{x(x + a + 1)\}\{(x + 1)(x + a)\} = a^2$

$$\text{or } \{x^2 + (a + 1)x\}\{x^2 + (a + 1)x + a\} = a^2$$

$$\therefore \{x^2 + (a + 1)x\}^2 + a\{x^2 + (a + 1)x\} - a^2 = 0$$

$$\therefore \{x^2 + (a + 1)x\}x = \frac{-a \pm \sqrt{a^2 + 4a^2}}{2} = \frac{-a \pm a\sqrt{5}}{2}.$$

$$\text{If } x^2 + (a + 1)x = \frac{-a + a\sqrt{5}}{2} \quad \dots (1)$$

$$\text{Then, } x^2 + (a + 1)x + \frac{a - a\sqrt{5}}{2} = 0$$

$$\text{If } x^2 + (a + 1)x = \frac{-a - a\sqrt{5}}{2}$$

$$\text{Then } x^2 + (a + 1)x + \frac{a + a\sqrt{5}}{2} = 0. \quad \dots (2)$$

If (1) and (2) have real roots then the given equation will have four real roots.

$\therefore$ the required conditions are as follows :

$$D_1 = (a + 1)^2 - 4 \cdot \frac{a - a\sqrt{5}}{2} \geq 0$$

$$\Rightarrow a^2 + 2a + 1 - 2(a - a\sqrt{5}) \geq 0$$

$$\Rightarrow a^2 + 2\sqrt{5}a + 1 \geq 0$$

$$\begin{aligned} \text{and } D_2 &= (a+1)^2 - 4 \cdot \frac{a+a\sqrt{5}}{2} \geq 0 \\ \Rightarrow a^2 + 2a + 1 - 2(a+a\sqrt{5}) &\geq 0 \\ \Rightarrow a^2 - 2\sqrt{5}a + 1 &\geq 0 \end{aligned}$$

Now, the required solution-set is the intersection of the solution-sets of

$$a^2 + 2\sqrt{5}a + 1 \geq 0 \quad \dots (3)$$

$$\text{and } a^2 - 2\sqrt{5}a + 1 \geq 0 \quad \dots (4)$$

For (3), the corresponding equation is

$$a^2 + 2\sqrt{5}a + 1 = 0.$$

$$\begin{aligned} \text{Roots are } a &= \frac{-2\sqrt{5} \pm \sqrt{20-4}}{2} \\ &= -\sqrt{5} \pm 2 \end{aligned}$$

$\therefore$ sign-scheme for $a^2 + 2\sqrt{5}a + 1, a \in R$ is as follows :

$$\begin{array}{c} (+) \qquad \qquad (-) \qquad \qquad (+) \\ \leftarrow \quad | \qquad \qquad | \qquad \qquad | \quad \rightarrow \\ \qquad -2-\sqrt{5} \qquad \qquad \qquad 2-\sqrt{5} \end{array}$$

$$\therefore a \leq -2 - \sqrt{5} \quad \text{or} \quad a \geq 2 - \sqrt{5}$$

For (4), the corresponding equation is $a^2 - 2\sqrt{5}a + 1 = 0$.

$$\begin{aligned} \text{Roots are } a &= \frac{2\sqrt{5} \pm \sqrt{20-4}}{2} \\ &= \sqrt{5} \pm 2 \end{aligned}$$

$\therefore$ sign-scheme for $a^2 - 2\sqrt{5}a + 1, a \in R$ is as follows :

$$\begin{array}{c} (+) \qquad \qquad (-) \qquad \qquad (+) \\ \leftarrow \quad | \qquad \qquad | \qquad \qquad | \quad \rightarrow \\ \qquad -2+\sqrt{5} \qquad \qquad \qquad 2+\sqrt{5} \end{array}$$

$$\therefore a \leq -2 + \sqrt{5} \quad \text{or} \quad a \geq 2 + \sqrt{5}$$

$\therefore$ taking the common values of a satisfying both (3) and (4),

$$a \leq -2 - \sqrt{5} \quad \text{or} \quad -\sqrt{5} + 2 \leq a \leq \sqrt{5} - 2 \quad \text{or} \quad a \geq \sqrt{5} + 2$$

$$\therefore \text{ the solution-set } = (-\infty, -2 - \sqrt{5}] \cup [2 - \sqrt{5}, -2 + \sqrt{5}] \cup [2 + \sqrt{5}, +\infty).$$

166. The logarithms will be defined if

$$x-2 > 0, x-2 \neq 1, x+2 > 0, x+2 \neq 1.$$

$$\text{Now, } \frac{\log 6}{\log(x-2)} + \frac{\log 6}{\log(x+2)} > \frac{\log 6}{\log(x-2)} \times \frac{\log 6}{\log(x+2)}$$

$$\text{or } \frac{\log(x+2) + \log(x-2)}{\log(x-2) \cdot \log(x+2)} > \frac{\log 6}{\log(x-2) \cdot \log(x+2)}$$

$$\text{or } \frac{\log(x^2 - 4) - \log 6}{\log(x-2) \cdot \log(x+2)} > 0 \quad \text{or} \quad \log_{10} \frac{x^2 - 4}{6} / [\log_{10}(x-2) \cdot \log_{10}(x+2)] > 0, \text{ taking the base 10.}$$

This is possible when any one of the following is true:

$$(i) \quad \log_{10} \frac{x^2 - 4}{6} > 0, \log_{10}(x - 2) > 0, \log_{10}(x + 2) > 0$$

$$(ii) \quad \log_{10} \frac{x^2 - 4}{6} > 0, \log_{10}(x - 2) < 0, \log_{10}(x + 2) < 0$$

$$(iii) \quad \log_{10} \frac{x^2 - 4}{6} < 0, \log_{10}(x - 2) > 0, \log_{10}(x + 2) < 0$$

$$(iv) \quad \log_{10} \frac{x^2 - 4}{6} < 0, \log_{10}(x - 2) < 0, \log_{10}(x + 2) > 0$$

$$(i) \quad \Rightarrow \quad \frac{x^2 - 4}{6} > 1, x - 2 > 1, x + 2 > 1$$

$$\Rightarrow \quad x^2 > 10, x > 3, x > -1$$

$$\Rightarrow \quad x > \sqrt{10}, \text{ for } x - 2 > 0 \text{ also.}$$

Similarly,

$$(ii) \quad \Rightarrow \quad x^2 < 10, x < 3, x < -1$$

$$\Rightarrow \quad x \text{ has no value because } x - 2 > 0 \text{ also.}$$

$$(iii) \quad \Rightarrow \quad x^2 < 10, x > 3, x < -1$$

$$\Rightarrow \quad x \text{ has no value.}$$

$$(iv) \quad \Rightarrow \quad x^2 < 10, x < 3, x > -1$$

$$\Rightarrow \quad 2 < x < 3 \text{ because } x - 2 > 0 \text{ also.}$$

Hence the solution-set = $(2, 3) \cup (\sqrt{10}, +\infty)$.

167. C.P = 1, 2, 3

$\therefore$ If $x < 1$, the inequation becomes $-(x - 1) - (x - 2) - (x - 3) \geq 6$

or $-3x + 6 \geq 6; \therefore x \leq 0$

$\therefore$ x is such that $x < 1$ and $x \leq 0$. So $x \leq 0$.

If $1 \leq x < 2$, the inequation becomes

$$(x - 1) - (x - 2) - (x - 3) \geq 6 \quad \text{or} \quad -x + 4 \geq 6; \therefore x \leq -2$$

No such values satisfy $1 \leq x < 2$.

If $2 \leq x < 3$, the inequation becomes $(x - 1) + (x - 2) - (x - 3) \geq 6$

or $x \geq 6$. No such values satisfy $2 \leq x < 3$.

If $x \geq 3$, the inequation becomes $(x - 1) + (x - 2) + (x - 3) \geq 6$

$$\text{or} \quad 3x - 6 \geq 6; \quad \text{or} \quad 3x \geq 12$$

$\therefore x \geq 4$ which satisfies $x \geq 3$.

$\therefore$ The values of x satisfy $x \leq 0$ or $x \geq 4$

$\therefore$ The required solution-set = $(-\infty, 0] \cup [4, +\infty)$.